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CHAPTER 14 RATIONAL EPRESSIONS

#### Introduction to Rational Expressions 14.1

## **Basic Concepts** • Simplifying Rational Expressions • Applications

## A LOOK INTO MATH >



Have you ever been moving smoothly in traffic, only to come to a sudden halt? Mathematics shows that in certain conditions, if the number of cars on a road increases even slightly, then the movement of traffic can slow dramatically. To understand why this occurs, we will consider how rational expressions can be used to model traffic flow. (See Example 6.)

# **Basic Concepts**

Recall that a *rational number* is any number that can be expressed as a ratio of two integers  $\frac{p}{q}$ , where  $q \neq 0$ . In this chapter, we discuss *rational expressions*, which can be written as the ratio of two polynomials. Because examples of polynomials include

3, 2x,  $x^2 + 4$ , and  $x^3 - 1$ ,

it follows that examples of rational expressions include

### **NEW VOCABULARY**

- □ Rational expression
- □ Lowest terms
- □ Vertical asymptote
- □ Probability

 $\frac{3}{2r}$ ,  $\frac{2x}{r^2+4}$ ,  $\frac{x^2+4}{3}$ , and  $\frac{x^3-1}{r^2+4}$ .

### **RATIONAL EXPRESSION**

A **rational expression** can be written as  $\frac{P}{Q}$ , where *P* and *Q* are polynomials. A rational expression is defined whenever  $Q \neq 0$ .

We can evaluate polynomials for different values of a variable. For example, for x = 2the polynomial  $x^2 - 3x + 1$  evaluates to

$$(2)^2 - 3(2) + 1 = -1.$$

Rational expressions can be evaluated similarly.

EXAMPLE 1

**TEACHING EXAMPLE 1** 

(b)  $\frac{t^2 - 1}{3t + 4}$ ; t = -2

ANS. (a)  $\frac{3}{2}$  (b)  $-\frac{3}{2}$ (c) Undefined (d) -1

the following. (a)  $\frac{3}{x-1}$ ; x = 3

Repeat Example 1 for each of

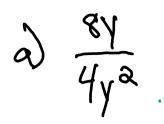
### **Evaluating rational expressions**

Use covers to create a work area.

If possible, evaluate each expression for the given value of the variable.

- (a)  $\frac{1}{x+1}$  x = 2 (b)  $\frac{y^2}{2y-1}$  y = -4(c)  $\frac{5z+8}{z^2-2z+1}$  z = 1 (d)  $\frac{2-x}{x-2}$  x = -3
- $\begin{array}{c} & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\$ (b) If y = -4, then  $\frac{y^2}{2y - 1} = \frac{(-4)^2}{2(-4) - 1} = -\frac{16}{9}$ . (c) If z = 1, then  $\frac{5z + 8}{z^2 - 2z + 1} = \frac{5(1) + 8}{1^2 - 2(1) + 1}$ , or  $\frac{13}{0}$ , which is undefined because division 7 = 1 by 0 is not possible
  - (d) If x = -3, then  $\frac{2-x}{x-2} = \frac{2-(-3)}{-3-2} = \frac{5}{-5} = -1$ .
  - Now Try Exercises 7, 11, 13, 17

Simpli



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b)  $\frac{2x+6}{3x+9}$