P859

14.1 Introduction to Rational Expressions

Basic Concepts • Simplifying Rational Expressions • Applications

A LOOK INTO MATH ▶

Have you ever been moving smoothly in traffic, only to come to a sudden halt? Mathematics shows that in certain conditions, if the number of cars on a road increases even slightly, then the movement of traffic can slow dramatically. To understand why this occurs, we will consider how *rational expressions* can be used to model traffic flow. (See Example 6.)

Basic Concepts

Recall that a *rational number* is any number that can be expressed as a ratio of two integers $\frac{p}{q}$, where $q \neq 0$. In this chapter, we discuss *rational expressions*, which can be written as the ratio of two polynomials. Because examples of polynomials include

$$3, \quad 2x, \quad x^2 + 4, \quad \text{and} \quad x^3 - 1,$$

it follows that examples of rational expressions include

$$\frac{3}{2x}, \quad \frac{2x}{x^2 + 4}, \quad \frac{x^2 + 4}{3}, \quad \text{and} \quad \frac{x^3 - 1}{x^2 + 4}.$$

NEW VOCABULARY

- ☐ Rational expression
- ☐ Lowest terms
- ☐ Vertical asymptote
- ☐ Probability

RATIONAL EXPRESSION

A **rational expression** can be written as $\frac{P}{Q}$, where P and Q are polynomials. A rational expression is defined whenever $Q \neq 0$.

We can evaluate polynomials for different values of a variable. For example, for $x = 2$ the polynomial $x^2 - 3x + 1$ evaluates to

$$(2)^2 - 3(2) + 1 = -1.$$

Rational expressions can be evaluated similarly.

EXAMPLE 1 **Evaluating rational expressions**

If possible, evaluate each expression for the given value of the variable.

(a) $\frac{1}{x + 1}$ $\quad x = 2$ **(b)** $\frac{y^2}{2y - 1}$ $\quad y = -4$

(c) $\frac{5z + 8}{z^2 - 2z + 1}$ $\quad z = 1$ **(d)** $\frac{2 - x}{x - 2}$ $\quad x = -3$

TEACHING EXAMPLE 1

Repeat Example 1 for each of the following.

(a) $\frac{3}{x - 1}$; $x = 3$

(b) $\frac{t^2 - 1}{\quad}$; $t = -2$

Use covers to create a work area.

a) $\frac{1}{x+1}$ $\quad x = 2$

b) $\frac{5z + 8}{z^2 - 2z + 1}$ $\quad z = 1$

Simplify

a) $\frac{8y}{4y^2}$

Insert blanks pages to create a work area.

b) $\frac{2x+6}{3x+9}$